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# Evolution of Harvesting Strategies

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# Evolution of harvesting strategies: replicator and resource dynamics

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## Abstract

Economic theories of managing renewable resources, such as fisheries and forestry, traditionally assume that individual harvesters are perfectly rational and thus able to compute the optimal harvesting strategy that maximizes their profits. The current paper presents an alternative approach based on bounded rationality and evolutionary mechanisms. It is assumed that individual harvesters face a choice between two harvesting strategies. The evolution of the distribution of strategies in the population is modeled through a replicator dynamics equation. The latter captures the idea that strategies yielding above average profits are more demanded than strategies yielding below average profits, so that the first type ends up accounting for a larger part in the population. From a mathematical perspective, the combination of resource and evolutionary processes leads to complex dynamics. The paper presents the existence and stability conditions for each steady-state of the system and analyzes dynamic paths to the equilibrium. In addition, effects of changes in prices are analyzed. A main result of the paper is that under certain conditions both strategies can survive in the long-run.

**Keywords:** Evolutionary economics – Renewable Resources – Common-Property Resource – Fishery.

**JEL-classification:** B25; Q30

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# 1 Introduction

The management of renewable resources, such as fish and timber, is traditionally studied within the neo-classical framework (Clark, 1976; Neher, 1990). In recent years, the growing criticism on this framework and the renaissance of evolutionary ideas applied to economics have given rise to new theoretical and modeling approaches. Underscoring the inadequacy between neoclassical assumptions and economic reality, these alternative approaches aim to solve some of the shortcomings of neoclassical theories. Two features have been subject to particular attention from evolutionary economists.

The first issue is related to the assumption of unboundedly rational choice. A standard result in resource economics is that the optimal harvesting rate over time can be computed by maximizing the present value of individual profits under the constraint of resource dynamics. This strongly implies that individual harvesters have perfect information and possess the computational abilities to solve the optimization problem. Nevertheless, this assumption has been refuted on empirical and methodological grounds (Conlisk, 1996; Simon, 1962). In recent years, bounded rationality has been blended within an evolutionary mechanism (Nelson and Winter, 1982). In this view, human behavior is guided by a set of ‘routines’, defined as rules of thumb or repeated habits. Accordingly, when new strategies or technologies are introduced on the market, firms tend to imitate the most successful one, or the one that yields a ‘satisficing’ level of profits. Thus, imitation and adaptation play an important role in individual choice. These notions are also relevant for the field of environmental economics (van den Bergh and Munda, 2000) and similar concepts, namely adaptive management and learning, have been introduced into environment-economic models (Holling, 1978; Walters, 1986).

The second issue deals with ‘representative agent’ models, as are common in mainstream economics. In resource economics, for instance, it is often assumed that the exploitation of a resource by a group of harvesters in non-free access situations is equivalent to a centrally planned economy (Levhari et al., 1981). According to Kirman (1992), these models are inaccurate or even misleading. In reality, agents can exhibit heterogeneous characteristics which interactions often lead to ‘emergent’ patterns that cannot be easily predicted at the population level by using representative agent models. In resource economics, the emergence of macro phenomena such as sustainability and pollution has been studied by Jager et al.

(2000) within a multi-agent model in which individuals are endowed with different behavioral characteristics. A similar type of model is used by Rouchier et al. (2001) to study emergent dynamics caused by the interactions among herdsmen managing a rangeland. In general, the use of such bottom-up approaches has strong implications for environmental policy. First, it emphasizes the role of self-regulation in ecological-economic systems. Second, it limits the role of environmental policy to interventions that aim at modifying the context in which agents interact. Accordingly, changing the context of interaction and allowing for self-regulation can result in diverse macro level outcomes that are not always predictable (Ostrom, 2000).

The current paper is a contribution to these new developments in economics and environmental economics. The objective is to study the interactions among a population of boundedly rational harvesters who compete for a single natural resource, and the macro level patterns that emerge from these interactions. This will have implications for resource use and management. Agents are boundedly rational in the sense that they are unable to compute the optimal level of effort that maximizes their profits. Instead, they choose between two predetermined harvesting strategies. Their decision for one strategy as opposed to the other is guided by a simple rule: namely, the harvesters tend to adopt the strategy that yields above-average profits. This behavior supposes that the agents receive imperfect information about strategies and profits of other harvesters and exhibit a certain degree of rationality, although limited.

This type of behavior, harvesters facing a choice between two heterogeneous strategies, corresponds to real-world situations. The ‘tuna war’ that opposed French and Spanish fishermen in the mid-90s in the Gulf of Biscay is a clear recent example. The Spanish resorted to traditional fishing gears, while the French used long drift nets and pelagic dragging, resulting in different productivity rates. In addition to technological characteristics, the harvesting strategies can also differ with respect to the type of information structure used by the different groups of fishermen. In North America, for instance, some groups of fishermen use Geographic Information Systems to systematically gather information on fish stocks, while other groups make use of less advanced methods of information processing and exchange. Ostrom (1990, 1994, 2000) in studies of common-dilemmas, i.e. issues where individual interest conflicts with the collective one, documents cases in which harvesters realise higher productivity through various cooperative strategies.

In our model, heterogeneity in strategies is expressed through different effort levels between two (sub)groups of harvesters. The strategy with the highest effort level can be associated with using a more productive technology or practices of information sharing and cooperation behavior. The latter interpretation is consistent with the low-effort level strategy following from ‘isolated harvesters’, i.e. those which do not cooperate or share information (about spatial and temporal features of the resource, such as in fisheries). Note that in our model, in contrast with resource games (Ostrom, 2000), a cooperative strategy enhances productivity and thus leads to higher harvest levels.

An approach that relates closely to the one developed in this paper is the analysis of common-dilemmas within an evolutionary game theory framework by Sethi and Somanathan (1996). They study the evolution of three strategies, namely ‘defecting’, ‘cooperating’, and ‘enforcing’ in the context of a population of harvesters interacting in a common-pool resource game. In such a game, individual payoffs are directly affected by the actions of the other players in terms of harvest levels and punishment. The evolutionary process is described by a replicator dynamics equation. The latter captures the idea that differential survival is proportional to the relative performance value of each strategy, where performance is here defined in terms of profits. Sethi and Somanathan integrate replicator and resource dynamics within a single framework and identify steady-states of the system, composed mainly of only ‘defectors’ or of coexistence of ‘cooperators’ and ‘enforcers’. Their analysis confirms that punishment can stimulate cooperation and is even necessary for solving common-dilemmas issues through spontaneous evolution of rules of thumb.

In dealing with the problem of selection of strategies by boundedly rational agents, several questions arise. First, what is the motivation behind the selection of one strategy over the other? In our model, selection is related to the performance, defined in terms of profits, of each strategy over time. Agents imitate the most successful strategy, and the adjustment speed at which harvesters switch to the superior strategy reflects the imperfect diffusion of information. We resort to a replicator dynamic equation to model the distribution of strategies in the population.

Second, what are the factors or dynamics that influence the performance of each strategy over time? Just like in Sethi and Somanathan’s model, we combine replicator and resource dynamics. Nevertheless, in our case individual payoffs are not directly affected by the aggregate behavior of the other harvesters.

Here individual choice for one strategy over another affects the stock of the common-pool resource, which in return determines individual's payoffs. Hence, the behavior of the group affects indirectly individual decisions by means of the resource stock. In this sense, our model does not describe a strategic game between the economic actors and is then by definition applicable to larger groups of harvesters.

Finally, what will be the long run distribution of strategies in the population? Here, we identify three evolutionary outcomes. In two cases, only one strategy survives while in a third case the two strategies coexist. The resource price appears to be a possible instrument of environmental policy.

The paper is structured as follows. Section 2 presents the model. Section 3 identifies the steady-states of the system. Section 4 presents the existence and stability conditions for each equilibrium. Section 5 discusses the dynamic path to the equilibrium using numerical illustrations. Section 6 presents the conclusions.

## 2 The model

We consider a system where a fixed number of individuals harvest a stock  $N$  of a natural resource. The natural growth dynamics governing the resource are described by a logistic equation. The stock is depleted by the aggregate harvest over all individuals in the population  $H(N)$ . Hence,

$$\dot{N} = rN\left(1 - \frac{N}{K}\right) - H(N) \quad (1)$$

where  $r$  is the intrinsic growth rate of the resource and  $K$  is the carrying capacity. We assume that the agents face a choice between two harvesting strategies. Each strategy is defined by a set of two parameters, namely an effort level  $E_i$  and an extraction cost parameter  $C_i$ , ( $i = 1, 2$ ). In traditional resource economics, the level of effort, defined in terms of labor or capital inputs, is the result of an optimization problem, whereas here the level of effort is determined exogeneously. In the remainder of the paper, we pose the assumption  $E_1 > E_2$  without loss of generality, meaning that the first strategy  $i = 1$  is more productive but also more resource intensive than the second one. As discussed in Section



1, differences in parameters reflect the idea that strategies differ with regard to acquiring of information, cooperation among agents, or use of technology.

In the present model, the harvest level  $h_i$  of an agent using strategy  $i$  ( $i = 1, 2$ ) is a linear function of the stock level  $N$ :

$$h_i(N) = E_i N \quad i = 1, 2 \quad (2)$$

Thus, the profits yielded by an agent using strategy  $i$  can be written as

$$\pi_i = h_i(N) \left[ P - \frac{C_i}{N+1} \right] \quad i = 1, 2 \quad (3)$$

where  $P$  is the exogenous price. The unit cost function  $\frac{C_i}{N+1}$  is commonly used in resource economics, and more particularly in fishery economics (Bradley, 1970). It reflects the idea that, as the stock of resource decreases, it becomes more and more costly to exploit it. In the remaining part of this paper, we aim to study how resource dynamics affect the level of profits of each strategy, and more particularly the ranking of profits between the two strategies, since it is this one that determines the evolution of strategies in the population. Note that the ranking of profits is not affected under the condition  $E_1 C_1 < E_2 C_2$ .<sup>1</sup> From (3), it appears that unit costs are positively affected by the product  $E_i C_i$ . Thus, posing  $E_1 C_1 > E_2 C_2$  implies that the total costs of strategy 1 exceed the total costs of strategy 2 for all  $N$ . This counterbalances the advantage of superior revenues of strategy 1, which is the most productive strategy. Indeed, as  $N$  is more and more depleted, total costs for strategy 1 increase more sharply than for strategy 2. As  $N$  keeps on decreasing, the difference between revenues and costs narrows for both strategies and eventually profits of strategy 1 fall below profits of strategy 2. Thus, further on we will assume that  $E_1 C_1 > E_2 C_2$ , since it corresponds to the interesting case where the outcome of the evolution of strategies cannot be easily

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<sup>1</sup>In this case, we have  $\pi_1 > \pi_2$  for all  $N$  in the following expression

$$\pi_1 - \pi_2 = PN(E_1 - E_2) - \frac{N}{N+1}(E_1 C_1 - E_2 C_2)$$

predicted without formal analysis.

We assume that there is a fixed number  $m$  of individual harvesters exploiting the resource. At any point in time, the proportion of agents using the strategy  $i = 1, 2$  is represented by the share  $s_i \in [0; 1]$ , where  $s_1 + s_2 = 1$ . Aggregate harvest can then be expressed as

$$H(N) = ms_1h_1(N) + m(1 - s_1)h_2(N) \quad (4)$$

The evolution of the proportion of individuals choosing strategy 1 is modeled through a replicator dynamics equation written as follows:

$$\dot{s}_1 = s_1(\pi_1 - \bar{\pi}) \quad (5)$$

with  $\bar{\pi} = s_1\pi_1 + (1 - s_1)\pi_2$  representing the average profit in the population. This equation captures the notion that a strategy yielding above (below) average profits increases (decreases) in share in the population. It is assumed that switching from one strategy to the other can be done at no costs. Two observations follow from (5). First, the change in shares is a gradual process. Therefore, an underlying assumption is that there must be a delay between observing that the other strategy is more profitable and adopting it. This can be interpreted as an element of imperfect information. For example, a more successful strategy may be employed somewhere outside the observation horizon of an agent. In brief, strategy changes are regarded as a result from imperfect learning about the performance of the other strategy, in other words as imitation. Second, the replicator dynamics in (5) implies that for  $s_1(0) = 1$  ( $s_1(0) = 0$ ), we have  $\bar{\pi} = \pi_1$  ( $\bar{\pi} = \pi_2$ ). In this case, the share  $s_1$  will remain constant over time. Since we aim to observe the evolution of strategies in the population, these cases are of no interest for our analysis. Thus, we assume that the population has an initial distribution characterized by  $0 < s_1(0) < 1$ .

For ease of interpretation, the general form of the replicator dynamics can be rewritten as follows since there are only two strategies:

$$\dot{s}_1 = s_1(1 - s_1)(\pi_1 - \pi_2) \quad (6)$$

In (6), when the profits of strategy 1 exceed the profits of strategy 2, the change in the proportion of individuals using strategy 1 is positive. Inversely, when  $\pi_2 > \pi_1$ , then  $(1 - s_1)$ , i.e. the share of strategy 2, increases. Equations (1) and (6) describe the full dynamic system. Through substitution of (2), (3) and (4) this reduces to

$$\begin{cases} \dot{N} = rN(1 - \frac{N}{K}) - mN(s_1E_1 + (1 - s_1)E_2) \\ \dot{s}_1 = s_1(1 - s_1)N[P(E_1 - E_2) + \frac{E_2C_2 - E_1C_1}{N+1}] \end{cases} \quad (7)$$

The structure of this system clearly shows that the dynamics of harvesting strategies are linked to the dynamics of the resource. At all points in time, the level of profits yielded by each strategy is determined by the stock level, and conversely, the resource level is determined by the distribution of harvesting strategies. The following section identifies the fixed points of the system.

### 3 Identification of the Steady-States

The steady states of the dynamical system (7) are situated at the intersections of the nullclides  $\dot{N} = 0$  and  $\dot{s}_1 = 0$ . We find  $\dot{N} = 0$  along the lines  $N = 0$  and  $s_1 = f(N)$ , given by

$$f(N) = \frac{r(1 - \frac{N}{K}) - mE_2}{m(E_1 - E_2)} \quad (8)$$

with  $f(0) = \frac{r - E_2m}{m(E_1 - E_2)}$ , and  $f(N) = 0$  for  $N = K(1 - \frac{mE_2}{r})$ . Next,  $\dot{s}_1 = 0$  for  $N = 0$ ,  $s_1 = 0$ ,  $s_1 = 1$ , and  $N = \frac{E_1C_1 - E_2C_2}{P(E_1 - E_2)} - 1$ . Figure 1 provides a graphical representation of the system. The set of equilibria is composed of three intersection points labelled A, B and C and of a set of equilibria  $\mathcal{D}$  along the line  $N = 0$ . Note that Figure 1 represents a special case where the three equilibria A, B and C lie within the feasible ranges for  $N$  and  $s_1$  i.e.,  $N \geq 0$  and  $0 \leq s_1 \leq 1$ . Section 4 will review the explicit existence conditions for each equilibrium.

[Insert Figure 1 here]

Solving the system (7) gives the coordinates of the three equilibria A, B and C and of the continuum of equilibria  $\mathcal{D}$ .

$$\begin{aligned} \text{A. } (N_A; s_{1A}) \text{ with } & N_A = K(1 - \frac{mE_2}{r}) \\ & s_{1A} = 0 \end{aligned}$$

$$\begin{aligned} \text{B. } (N_B; s_{1B}) \text{ with } & N_B = \frac{E_1C_1 - E_2C_2}{P(E_1 - E_2)} - 1 \\ & s_{1B} = \frac{P(E_1 - E_2)[r(K+1) - mKE_2] + r(E_2C_2 - E_1C_1)}{mKP(E_1 - E_2)^2} \end{aligned}$$

$$\begin{aligned} \text{C. } (N_C; s_{1C}) \text{ with } & N_C = K(1 - \frac{mE_1}{r}) \\ & s_{1C} = 1 \end{aligned}$$

D. The set  $\mathcal{D}$  of all points such that  $N = 0$ . The set  $\mathcal{D}$  corresponds to a degenerate equilibrium in which there is no harvesting activity at all. Accordingly, we will exclude the set  $\mathcal{D}$  from further analysis, since differences among strategies in this set are irrelevant.

A brief look at the steady-states A, B and C shows that there are three possible distributions of strategies over the population in the long run. In the A-equilibrium only the least resource intensive strategy survives, meaning that ultimately all harvesters have adopted strategy 2. In the C-equilibrium, by contrast, strategy 1 is the sole strategy that remains in the population. Remarkably, in the B-equilibrium both strategies coexist in the long-run, meaning that competition between strategies does not necessarily lead to the elimination of one of them.

Two central observations can be made about the B-equilibrium. First, the B-equilibrium corresponds to a situation where both strategies yield equal profits. Indeed, at  $N = N_B$  the effort level differences cancel out against the costs. For  $N < N_B$ , profits of the least productive strategy 2 exceed profits of the most productive one, strategy 1. Consequently, harvesters switch progressively to strategy 2. Inversely, the ranking of profits alternates when  $N$  increases beyond  $N_B$ . Then, for  $N > N_B$  the profits of strategy 1 exceed the profits of strategy 2, leading to a gradual increase in the share  $s_1$ .

A second observation is that the position of the B-equilibrium is affected by changes in the price vari-

able. From the coordinates of  $B$ , it can be seen that  $N_B(s_{1B})$  is a decreasing (increasing) function of  $P$ . Effects of a positive shift in the price  $P$  are depicted in Figure 2 and 3.

[Insert Figure 2 here]

[Insert Figure 3 here]

A positive change in price from  $P_1$  to  $P_2$  corresponds to a shift to the left of the line  $N = N_B$  in Figure 3. This results into a lower stock level and a higher share of harvesters using the most productive strategy 1 at the B-equilibrium.

## 4 Existence and stability conditions

### 4.1 Existence conditions

This section specifies conditions for the existence of each equilibrium.

- Existence of the A-equilibrium.

A necessary condition for the existence of the A-equilibrium is  $N_A > 0$ . This holds if and only if  $r > mE_2$ . In other words, the intrinsic growth rate of the resource must exceed the total harvest rate when the whole population uses the least resource intensive strategy, namely strategy 2.

Note that for  $r \leq mE_2$ , there is complete exhaustion of the resource at all equilibria, meaning that unsustainability is inevitable.

- Existence of the C-equilibrium

A necessary condition for the existence of the C-equilibrium is  $N_C > 0$ . This holds if and only if  $r > mE_1$ . In other words, the intrinsic rate of growth of the resource is larger than the total harvest rate when all harvesters use the most intensive strategy, namely strategy 1.

Note that  $r \leq mE_1$  is a necessary condition for possible exhaustion of the resource. It implies that

the resource stock at C is negative. Nevertheless, sustainability can be achieved at the remainder equilibria A and B. Further on, we will exclude the possibility of resource exhaustion and assume  $r > mE_1$ .

- Existence of the B-equilibrium.

Necessary conditions for the existence of the B-equilibrium are  $N_B > 0$  and  $0 \leq s_{1B} \leq 1$ . The first condition is satisfied whenever

$$P < \frac{E_1 C_1 - E_2 C_2}{E_1 - E_2} \quad (9)$$

The second condition  $0 \leq s_{1B} \leq 1$  implies that the price  $P$  must be set within a specific range, namely

$$P^* < P < P^{**} \quad (10)$$

where  $P^*$  and  $P^{**}$  are defined as follows

$$P^* = \frac{r(E_1 C_1 - E_2 C_2)}{[r(K+1) - mK E_2](E_1 - E_2)} \quad (11)$$

$$P^{**} = \frac{r(E_1 C_1 - E_2 C_2)}{[r(K+1) - mK E_1](E_1 - E_2)} \quad (12)$$

Note that for  $P = P^*$  ( $P = P^{**}$ ), the B-equilibrium corresponds exactly to the steady-state A (C). Under these equality conditions only one strategy survives in the long-run. In addition, it can be noted that since we abstract from resource exhaustion, i.e. we assumed  $r > mE_1$ , then both prices  $P^*$  and  $P^{**}$  are positive and satisfy condition (9), so that (10) is a sufficient condition for the existence of the B-equilibrium. In brief, the existence condition for B is fully given by (10), which is equivalent of saying that  $N_B$  must be located between  $N_A$  and  $N_C$ .

In conclusion, depending on the parameter conditions, the model may display either one unique equilibrium or multiple equilibria.

## 4.2 Stability results

Figure 1 provides a qualitative framework to analyze the global stability properties of the system. The arrows indicate the direction of movement of partial derivatives of  $N$  and  $s_1$  with  $\dot{N} < 0$  for  $s_1 > f(N)$ , and  $\dot{s}_1 < 0$  for  $N < N_B$ . In the following, we present the global stability results corresponding to each equilibrium point.

### Proposition 1 Global stability of the A-equilibrium

*The equilibrium A is globally asymptotically stable if  $P \leq P^*$  and  $r > mE_2$ .*

*Proof: see Appendix A.1.*

Proposition 1 tells us that the dynamic process drives the strategy 1 out of the population such that in the long-run only strategy 2 survives. Note that if the condition  $r > mE_2$  is not satisfied then  $N_A < 0$ , and the A-equilibrium cannot be reached.

For A to be globally stable, we must have  $P < P^*$ . As shown in Section 3, the value of the price affects the position of the B-equilibrium. Thus  $P < P^*$  corresponds to the condition  $N_B > N_A$ , meaning that the B-equilibrium is located on the right of the steady-state A in Figure 1 and conditions for existence of B are not satisfied. In this situation, both equilibria A and C are located on the left of B, where profits of B are not satisfied. The only stable equilibrium is thus the steady-state A where all harvesters have ultimately adopted strategy 2. Thus, as soon as the resource level is depleted beyond  $N_B$ , the dynamical system converges to A.

### Proposition 2 Global stability of the C-equilibrium

*The equilibrium C is globally asymptotically stable if  $P \geq P^{**}$  and  $r > mE_1$ .*

*Proof: see Appendix A.2.*

In this case, the dynamical process drives strategy 2 out of the population such that in the long-run only strategy 1 survives. If the condition  $r > mE_1$  is not satisfied, then  $N_C < 0$  and the C-equilibrium cannot

be reached.

The condition  $P > P^{**}$  implies  $N_B < N_C$ , meaning that the B-equilibrium is located on the left of the steady-state C in Figure 1. In this situation, both equilibria A and C are on the right of  $N_B$ , where profits of strategy 1 exceed profits of strategy 2. Thus, all harvesters will progressively switch to strategy 1, meaning that the C-equilibrium is the only asymptotically stable equilibrium.

**Proposition 3 Global stability of the B-equilibrium**

*The equilibrium B is globally asymptotically stable if it exists. For a price far enough from  $P^*$  and  $P^{**}$ , the trajectory of the dynamical system spirals inward towards the point B.*

*Proof: see Appendix A.3.*

When the value of  $P$  lies outside the range defined by  $P^*$  and  $P^{**}$ , the B-equilibrium cannot be reached. When B exists, the trajectory is a stable focus that converges to B. Note that there are values of  $P$  very close to  $P^*$  and  $P^{**}$  for which there is no spiralling around B (see Appendix A.3). Fluctuations around B are caused by alternances in the ranking of profits. These occur whenever the dynamical process crosses the line  $N = N_B$ . Asymptotic stability of B is illustrated in more details in Section 5.

## 5 Dynamic paths to the equilibrium: Numerical illustration

In this section, we provide an illustration of stability at the B-equilibrium. As stated earlier, in this case both strategies coexist in the long run. The following analysis can then be easily transposed to stability at the equilibria A and C. We assume  $C_1 > C_2$ , so that the most productive strategy exhibits higher costs. We fix the parameters as follows:  $E_1 = 0.7$ ,  $E_2 = 0.2$ ,  $C_1 = 15$ ,  $C_2 = 10$ ,  $r = 8$ ,  $K = 50$ ,  $m = 10$ ,  $N(0) = 50$ ,  $s_1(0) = 0.4$ . Note that  $P^* = 0.44$  and  $P^{**} = 2.34$ , so we fix  $P = 1$  implying that the existence condition for B is satisfied. In this case, B is globally asymptotically stable as stated in Proposition 3. The coordinates of the fixed points are  $A(37.5; 0)$ ,  $B(16; 0.68)$ ,  $C(6.25; 1)$ . The existence conditions are satisfied for all equilibria and the dynamical process converges towards the point B as shown in Figure 4.



[Insert Figure 4 here]

To provide a better understanding of the processes involved, Figure 5 presents the evolution over time of the stock of resource, the share  $s_1$  and the profits yielded by both strategies.

[Insert Figure 5 here]

In Figure 5, it can be observed that  $s_1$  first increases over time. Simultaneously,  $N$  decreases since an increasing number of harvesters progressively adopts strategy 1. When the resource level reaches  $N_B = 16$ , the share  $s_1$  starts decreasing. This is due to the fact that at  $N_B$  the ranking of profits alterns and the profits yielded by strategy 1 become inferior to the profits yielded by strategy 2, as it can be seen on the lower graph of Figure 5. Consequently,  $s_1$  continues to decrease as long as  $\pi_2 > \pi_1$ . Nevertheless, there is a point where the dynamic trajectory crosses the line  $s_1 = f(N)$  in Figure 4. When the trajectory goes below the line  $s_1 = f(N)$ , we have  $\dot{N} > 0$ , meaning that the resource starts to replenish itself. Thus, the resource stock increases until reaching  $N_B = 16$ . At that point, the ranking of profits alterns again as shown on the bottom part of Figure 5, and  $s_1$  starts increasing one more time. The process continues and forms a spiral that converges toward the point B where the process ends.

Note that in this example, we assume that the harvesters cannot exit the industry even when facing negative profits. This assumption is realistic since fisheries, for instance, are characterized by ease of entry but difficulty to exit. When prices or yields drop, there are no alternative opportunities for using the vessels. Thus, the managers often remain in the fishery as long as they are able to cover their direct operating costs.

In addition, it can be noted that for each strategy there is a specific  $N$  below which profits become negative. Whether the dynamical process reaches this threshold or not depends on the initial proportion of high harvesters in the population. Indeed, when the population is initially composed of very few high harvesters, subsequent harvests remain low and the stock level is likely to stay above the threshold level. By contrast, when the population is initially composed of a large number of high harvesters, the resource stock is immediately harvested beyond the threshold level below which profits become negative.

## 6 Discussion and conclusion

The current paper is a contribution to the literature on evolutionary economics applied to environmental issues. Two features make the model analyzed innovative.

First, in the spirit of evolutionary economics theories, we considered boundedly rational agents. Imitation is a central feature of the model and can be seen as imperfect learning, since we assumed that information about successful strategies diffuses imperfectly. Harvesters in our model must evaluate the performance of their strategy at each point in time, and may then decide to switch to a superior strategy. Superiority is not fixed but changes over time, influenced by the resource stock and the distribution of strategies in the population.

Second, the model combines interactive resource and replicator dynamics in a single framework. The evolution of harvesting strategies modifies the level of the resource stock. Inversely, changes in resource levels affect the profits and thus the evolution of economic activities. Individual profits are solely affected by the resource stock level, and the latter is directly linked to the aggregate behavior of harvesters.

Our analysis identifies three evolutionary outcomes or in other words three possible configurations of the final population state. In two cases, the whole population adopts a unique strategy, either the high or low effort one. In the third case, the population of harvesters is distributed between the two strategies. We showed how price changes affect existence and discussed the stability of each of those outcomes.

In our model, we used the price as the instrument of environmental policy. A price set too low or too high will ultimately result in a dominance of one strategy. A mid-range price allows for coexistence of the two strategies. An increase in price causes more and more harvesters to switch to the most productive strategy, namely strategy 1. Recall that in the model high harvesters are penalized by higher unit costs, so that there is a certain level of the resource stock at which profits of strategy 1 become inferior to profits of strategy 2. Thus, increasing the price results in lowering the threshold resource stock below which strategy 1 loses its advantage in terms of profits. The price can here be seen as a contextual variable. Changing the context in which agents interact disturbs the equilibrium and can result in a different evolutionary

outcome. Nevertheless, other variables affecting resource parameters for instance could also play this role and lead to similar results.

Which equilibrium is desirable in the long-run depends upon policy objectives. As stated, the achievement of sustainability is related to parameter settings. If the growth rate of the resource is below the total harvest rate when the whole population uses the least resource intensive strategy 2, then unsustainability is inevitable. In all other cases, the possibility of exhaustion of the system depends on existence conditions. In terms of resource conservation, the point A is the equilibrium where the highest resource level can be achieved in the long run. This makes sense intuitively since at A there are no harvesters using the highly resource intensive strategy. Finally, whenever policy aims at preserving diversity of strategies, the B-equilibrium is the most desirable outcome to achieve, since at B, there is coexistence of the two competing strategies. Preserving diversity of harvesting strategies can be relevant to allow adaptive flexibility in the face of system uncertainty, or more simply to conserve knowledge in the system. In addition, heterogeneity may also be desirable for social or political reasons. When applied to fisheries for instance, this model shows that cohabitation is possible between artisanal and industrial fisheries on the same fishing grounds.

# APPENDIX A - Proof of global stability results

## Appendix A.1: Proof of proposition 1.

In Figure 1, the arrows indicate the signs of the derivatives:  $s_1$  is positive (negative) for  $N > N_B$  ( $N < N_B$ ). In addition,  $\dot{N}$  is positive (negative) for all points below (above) the curve  $s_1 = f(N)$ . In the following sections, we present the global stability results that correspond to each equilibrium.

Consider the situation  $N_B \geq N_A$  as depicted in Figure 6. For  $N > N_B$ ,  $s_1$  increases monotonically, and  $N$  decreases since  $N > N_B$  corresponds to a point located above the curve  $s_1 = f(N)$ . However, the increase of  $s_1$  cannot be maintained until  $s_1 = 1$ . This can be seen by solving the differential equation for  $\dot{s}_1$ .

$$\dot{s}_1 = s_1(1 - s_1)N[P(E_1 - E_2) + \frac{E_2C_2 - E_1C_1}{N + 1}] \quad (\text{A-1})$$

which can be rewritten as

$$\dot{s}_1 = as_1(1 - s_1) \quad (\text{A-2})$$

where  $a$  is a positive constant. The solution of the differential equation is

$$s_1(t) = \frac{1}{(\frac{1}{s_1(0)} - 1)e^{-at} + 1} \quad (\text{A-3})$$

Note that  $s_1(t) \rightarrow 1$  requires  $t \rightarrow \infty$ . In other words,  $s_1$  will never reaches the value  $s_1 = 1$ . This implies that  $N$  will fall below  $N_B$  before  $s_1$  reaches 1, after which  $s_1$  starts decreasing.

In a first step,  $N$  continues to decrease until  $s_1$  passes below the curve  $s_1 = f(N)$ . Note that if the trajectory were to reach  $s_1 = 1$  or  $s_1 = 0$ , the value of  $s_1$  would remain constant over time.

In addition, the dynamical system cannot reach  $N = 0$  as long as the condition  $r > mE_1$  holds. The

growth rate of the resource is expressed as

$$\frac{\dot{N}}{N} = -\frac{rN}{K} - ms_1(E_1 - E_2) + r - mE_2 \quad (\text{A-4})$$

Indeed,  $r > mE_1$  implies that  $\frac{\dot{N}}{N}$  is increasing for all  $N > 0$  and  $0 \leq s_1 \leq 1$ . Note that, whenever  $r \leq mE_1$ ,  $\frac{\dot{N}}{N}$  decreases for  $N$  close to 0 and  $s_1$  close to 1, meaning that the process would ultimately reach  $N = 0$ .

In a second step, the trajectory crosses  $s_1 = f(N)$  such that  $N$  increases and  $s_1$  decreases until reaching A. The process ends in A where  $s_1 = 0$ .

[Insert Figure 6 here]

## Appendix A.2: Proof of Proposition 2.

Consider the case  $N_B \leq N_C$  as depicted in Figure 7. For  $N < N_B$ ,  $s_1$  decreases. Nevertheless,  $s_1$  cannot reach  $s_1 = 0$ . Indeed, according to the equation (A-3),  $s_1(t) \rightarrow 0$  requires  $t \rightarrow \infty$ . As  $N < N_B$  corresponds to a point below the curve  $s_1 = f(N)$ ,  $N$  increases. When  $N$  reaches the stock level  $N_B$ ,  $s_1$  starts increasing, and accordingly when  $s_1$  moves above the curve  $s_1 = f(N)$ ,  $N$  starts decreasing. Ultimately,  $s_1$  increases monotonically until reaching the equilibrium C where  $s_1 = 1$ . The process approaches the point C asymptotically to the line  $s_1 = 1$  and ends in C where  $s_1 = 1$ .

[Insert Figure 7 here]

## Appendix A.3: Proof of Proposition 3.

When  $N_B$  is located between  $N_C$  and  $N_A$  as depicted in Figure 8, the trajectory follows an anti-clockwise move. As stated above,  $s_1$  will increase (decrease) without ever reaching  $s_1 = 1$  ( $s_1 = 0$ ), and similarly

$N$  will decrease without ever reaching  $N = 0$  and a continuous increase in  $N$  cannot be maintained over time since once  $N$  goes beyond the curve  $s_1 = f(N)$  it starts decreasing again. Thus, B is asymptotically globally stable.

In addition it can be shown that the trajectory may spiral around B before converging to it. The local stability results show that there exists a neighborhood near B where the trajectory is a spiral.

To prove that the point B is a stable focus, or in other words that the dynamics may exhibit a spiralling pattern when converging to B, we need to show that the eigenvalues of the Jacobian matrix are complex conjugates and that in addition the real parts are negative. The presence of a limit cycle would be characterized by pure imaginary eigenvalues. The Jacobian matrix of the linearized system (7) has the form

$$J = \begin{pmatrix} \frac{\partial \dot{N}}{\partial N} & \frac{\partial \dot{N}}{\partial s_1} \\ \frac{\partial \dot{s}_1}{\partial N} & \frac{\partial \dot{s}_1}{\partial s_1} \end{pmatrix} \quad (\text{A-5})$$

Estimated at B, the Jacobian becomes

$$J = \begin{pmatrix} -r \frac{N_B}{K} & -\alpha m N_B \\ s_{1B}(1 - s_{1B}) N_B \frac{\alpha^2 P^2}{\beta} & 0 \end{pmatrix} \quad (\text{A-6})$$

where we define  $\alpha = E_1 - E_2$  and  $\beta = E_1 C_1 - E_2 C_2$ . We observe a stable focus whenever for the following matrix  $J = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ , the conditions  $a^2 + 4bc < 0$  are  $a < 0$  are satisfied. For the Jacobian evaluated at the point B, the first condition is equivalent to

$$\frac{r^2 N_B^2}{K^2} - 4\alpha m N_B s_{1B}(1 - s_{1B}) N_B \frac{\alpha^2 P^2}{\beta} < 0 \quad (\text{A-7})$$

After substitution and simplification, the condition (A-7) becomes

$$4(N_A - N_B)(N_B - N_C) \frac{P^2 \alpha}{m \beta} > 1 \quad (\text{A-8})$$

The prices  $P$ ,  $P^*$  and  $P^{**}$  can be written as follows:

$$P = \frac{\beta}{\alpha(N_B+1)}, \quad P^* = \frac{\beta}{\alpha(N_A+1)}, \quad P^{**} = \frac{\beta}{\alpha(N_C+1)}$$

The condition (A-8) can be expressed as:

$$-4 \frac{\beta}{\alpha m} \left( \frac{P}{P^*} - 1 \right) \left( \frac{P}{P^{**}} - 1 \right) > 1 \quad (\text{A-9})$$

Recall that for  $E_1 > E_2$ , we have  $P^* < P < P^{**}$ . Thus, for  $P$  close to  $P^*$  or  $P$  close to  $P^{**}$ , the inequality (A-9) does not hold.

Finally, the second condition  $a < 0$  is verified since  $-r \frac{N_B}{K} < 0$  for  $N_B > 0$ . Note that for  $N_B = 0$ ,  $a = 0$  and the eigenvalues of the matrix evaluated at B are pure imaginary, meaning that we observe a limit cycle around B. This case is excluded from our analysis and the oscillations converge to B.

In conclusion, the trajectory of the dynamical system is spiralling under certain conditions, i.e. if the parameters are set such that the inequality (A-9) holds.

[Insert Figure 8 here]

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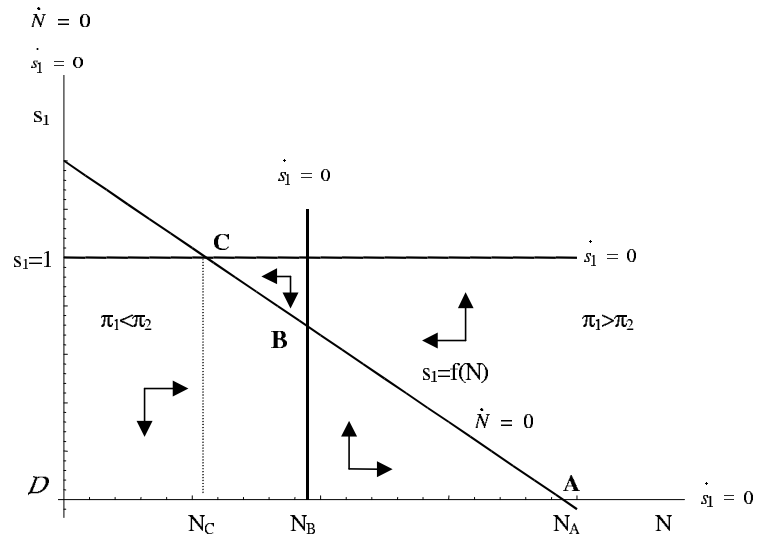


Figure 1: Graphical representation of the dynamic system

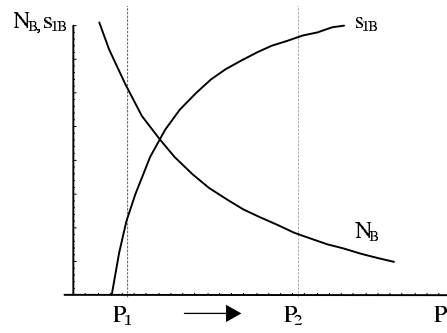


Figure 2:  $N_B(P)$  and  $s_{1B}(P)$

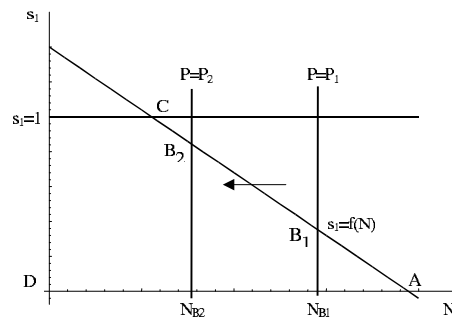


Figure 3: Shift in  $N_B$  due to a change in price



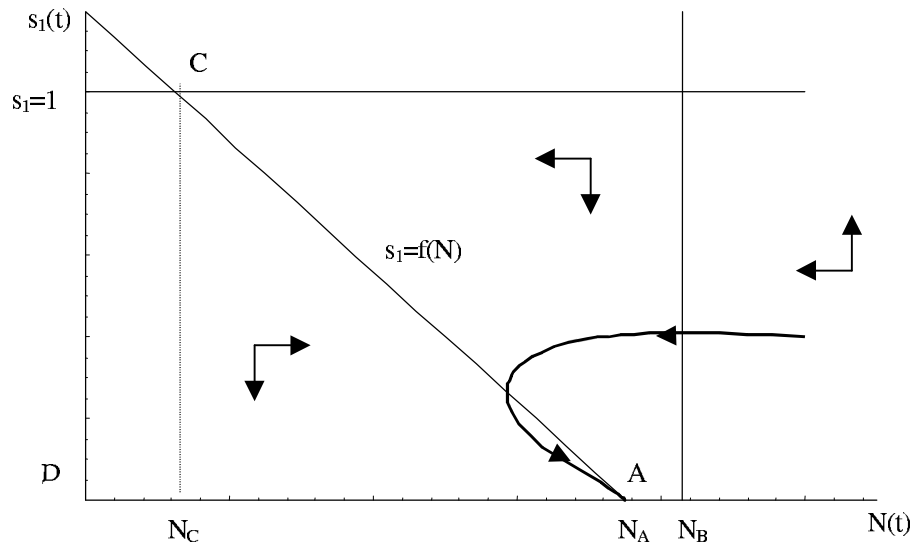


Figure 6: Global stability of the A-equilibrium

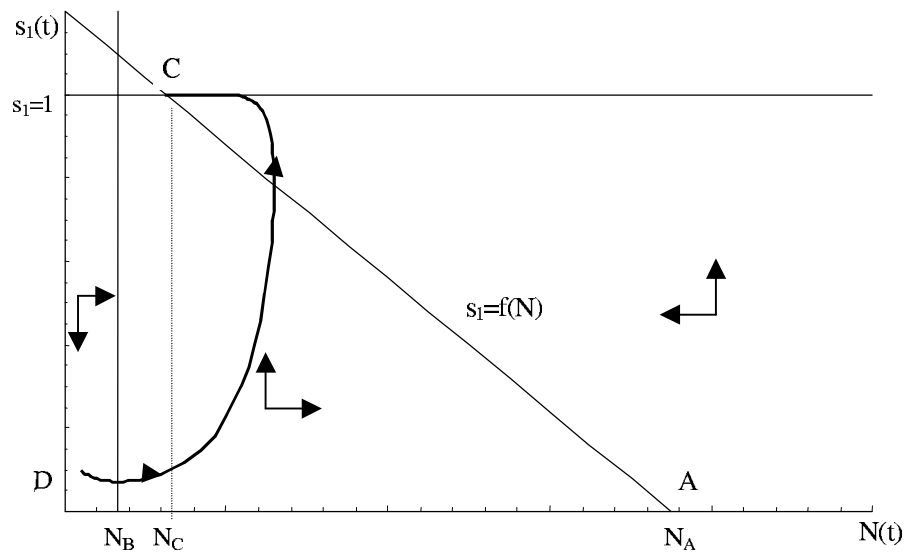


Figure 7: Global stability of the C-equilibrium

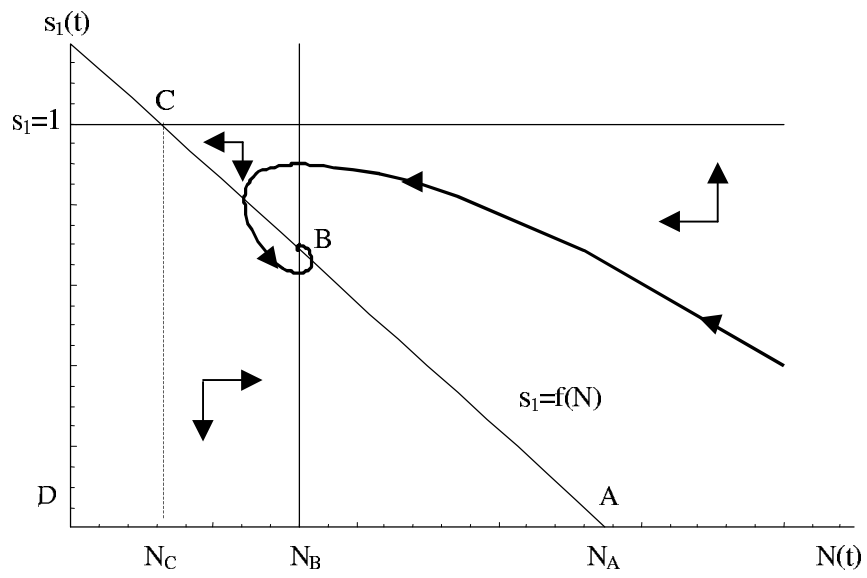


Figure 8: Global stability of the B-equilibrium